By R, I underſtand the Radius of the Circle Generant; and by s, the Right Sine of the Arch BA, whoſe Verſed Sine is VA.

And, where ever in my whole Diſcourſe of the *Cycloid*, or the Reſtored Trilinear (which is a Figure of Archs, and a Figure of Verſed Sines) the Arch a is no Ingredient in the deſignation; ſuch part or portion of them is capable of being Geometrically ſquared. But when I exclude a, I do therein exclude P (for that is an Arch alſo) and $f=a+s$, and $e=a-s$, becauſe a is therein included.

Mr. *Caſwell*, (not being aware that I had ſquared theſe Figures) had done the ſame by a Method of his own, (which he ſhewed me lately) which I would have inſerted here, but that he thought it not neceſſary; and inſtead thereof, hath given me the Quadrature of a Portion of the *Epicycloid* (which you will receive with this) and, I think, it is purely new.

IV. *The Quadrature of a Portion of the Epicycloid.* By Mr. Caſwel.

SUppoſe DPV to be half of an exterior Epicycloid, VB its Axis, V the Vertex, VLB half of the generant Circle, E its Center; DB the Baſe, C its Center: Biſect the Arc of the Semicircle VB in L, and on the Center C through L draw a Circle cutting the Epicycloid in P: Then I ſay the Curvilinear Triangle VLP will be $= BEq$ in $\frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle will be to the Curvilinear Triangle VLP, as CB the Semidiameter of the Baſe, to CE: which CE in the exterior Epicycloid is the

the Sum of the Semidiameters of the Base and Generant, but in the Interior Epicycloid Dpu, 'tis the difference of the said Semidiameters.

COROLLARY.

In the Interior Epicycloid, if CE is $\frac{1}{2}CB$, the Epicycloid then degenerating into a right Line, the Quadrature of the Triangle lpu will be in effect the same with the Quadrature of *Hippocrates Chius*.

COROL. II.

If the Semidiameter of the Base is supposed infinite, the Epicycloid then being the common Cycloid, the Area of the said Triangle will be equal to the Square of the Radius of the Generant, and so it falls in with that Theorem which *Lalovera* found, and calls *Mirabile*.

Though I do not think the abovesaid Quadrature can easily be deduced from what has been yet published of the Epicycloid, I have not added the Demonstration; but think it enough to name a general Proposition from whence I deduced it, *viz.* The Segments of the Generant Circle are to the Correspondent Segments of the Epicycloid, as CB to $2CE+CB$. For Example, suppose FmG the Position of part of the Generant when the point F of the Exterior Epicycloid was designed, then the Segment $FmGn$ is to the Segment $DFnG$: : as CB to $2CE+CB$.

And consequently the whole Epicycloid to the whole Generant in the same Proportion: Which is the only case demonstrated by Monsieur *De la Hire*.

It follows also that in the Vulgar Cycloid, its Segments are triple of the Correspondent Sectors of the Generant, which was first shewn by Dr. *Wallis*.

A Demonstration hereof, with a General Proposition for all Curves of this kind, shall be given in the next Transaction. V. A.

Fig. 2

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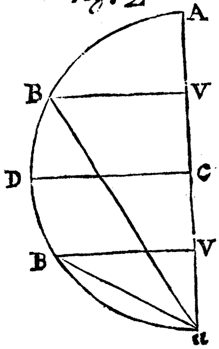
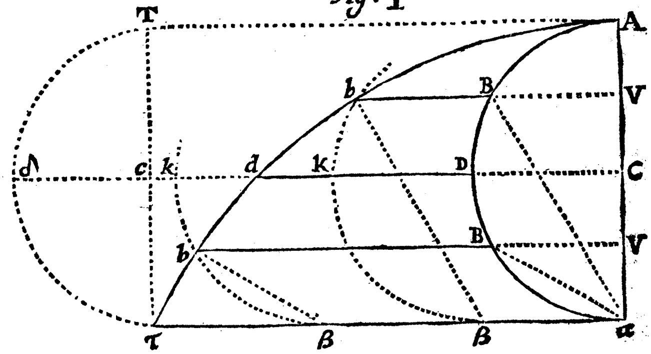


Fig. I



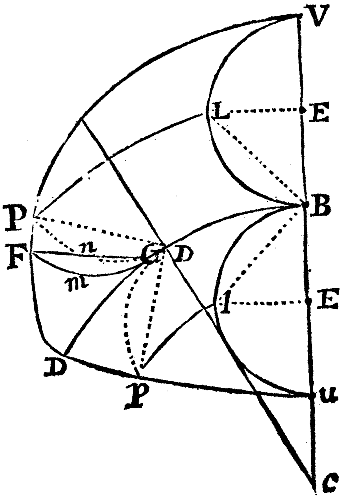


Fig. 3